The Longitudinal Mass (inertia) of a particle moving at Relativistic Speed

For a **stationary** laser/particle, let:

e_0 = The energy per unit length of the upstream/downstream wave.
L_0= The length of the laser's resonant cavity (or particle) at rest.
f_0 = The frequency of the upstream/downstream waves in the stationary laser.
E_0= The total energy of the waves comprising the particle (or of the of the laser's standing waves).
m_0= The rest mass of the particle (or mass equivalent of the laser's standing waves).

As there are two waves inside the laser's resonant cavity (upstream & downstream), the total energy of the waves inside the cavity is:

$$E_0 = 2 \times (e_0 \times L_0) \qquad (1)$$

The inertia of a standing-wave particle depends on the total sum of the momenta of the upstream and downstream waves (which are constantly moving at the speed of light and reflecting off the standing wave nodes).

For a laser (or particle) that is **moving** at Relativistic speed, the following equations apply:

Let e_{up} = energy per unit length of the upstream wave.
Let e_{down} = energy per unit length of the downstream wave.

The frequency of the upstream wave is:

$$f_{up} = f_0 \times \frac{c}{c-v} \qquad (2)$$

The frequency of the downstream wave is:

$$f_{down} = f_0 \times \frac{c}{c+v} \qquad (3)$$

As the energy per unit length of a wave is proportional to the wave's frequency ($e \propto f$), then (2) and (3) can be rewritten as:

The energy per unit length of the upstream wave is:

$$e_{up} = e_0 \times \frac{c}{c-v} \qquad (5)$$

The energy per unit length of the downstream wave is:

$$e_{down} = e_0 \times \frac{c}{c+v} \qquad (6)$$

The total energy per unit length:

$$E_{moving} = \left(e_0 \times \frac{c}{c-v} + e_0 \times \frac{c}{c+v}\right) \qquad (7)$$

The total energy per second:

$$E_{moving}c = \left(e_0 \times \frac{c^2}{c-v} + e_0 \times \frac{c^2}{c+v}\right) \qquad (8)$$

However, time is running slower by a factor of γ for the moving particle, which means it takes γ times longer for the standing waves in the particle to communicate the momentum change through the particle.

So:

$$E_{moving}c = \gamma\left(e_0 \times \frac{c^2}{c-v} + e_0 \times \frac{c^2}{c+v}\right) \qquad (9)$$

But $E = pc$, so:

$$p = \gamma e_0 \left(\frac{c}{c-v} + \frac{c}{c+v}\right) \tag{10}$$

$$p = \gamma e_0 \left(\frac{c(c+v)}{c^2 - v^2} + \frac{c(c-v)}{c^2 - v^2}\right)$$

$$p = \gamma e_0 \left(\frac{c^2 + cv}{c^2 - v^2} + \frac{c^2 - cv}{c^2 - v^2}\right)$$

$$p = \gamma e_0 \left(\frac{2c^2}{c^2-v^2}\right) \ = \ \gamma e_0 2\gamma^2 \ = \ 2e_0\gamma^3 \tag{11}$$

The total energy per unit length of stationary particle is:

$$E_{stationary} = 2e_0 c \tag{12}$$

Thus,

$$p_{stationary} = 2e_0 \tag{13}$$

So,

$$\frac{p}{p_{stationary}} = \frac{2e_0\gamma^3}{2e_0} = \gamma^3 \tag{14}$$

Therefore, the longitudinal mass m_L is:

$$m_L = \gamma^3 m_0 \tag{15}$$